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# ESTIMATING FUZZY HAZARD RATE OF THREE PARAMETERS WEIBULL DISTRIBUTION

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## Abstract

It is noticed there are many researches and studies in the field of statistics particularly in the field of hazard aiming in order to obtain estimators of very high standard of competency or tainted by instances of uncertainty, which often leads the researcher to estimate the fuzzy hazard functions instead of the usual hazard function, it is the most comprehensive and representative of the data for which the hazard function is to be applied. This paper deals with estimating of fuzzy hazard rate function of three parameters Weibull, methods of Moments Estimators (MOE), maximum likelihood estimators (MLE), and Regression method. The comparison is done by simulation method using different sample size (n=40,60, 80) and initial values of $(b,\ c, \delta)$(b: scale, c: shape , $\delta$: location parameter). after the parameters are estimated by different three methods (MLE, MOM, PEC(Regression) ; the values of ( $t_i$ ) is generated from C.D.F, using inverse transformation. set of five values of ( $t_i$ ) have been taken for application and estimation process. The goal of this process is to get the lowest value of the Mean square error, many fidelity criteria have been computed and the results have been discussed.

**Keywords:** Fuzzy hazard rate, Weibull Distribution, Maximum Likelihood Estimator (MLE), Moments estimator (MOM), Percentiles.

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## Introduction:

Weibull distribution received much interest in reliability theory, and it is used to describe real phenomena and modeling distribution of breaking strength of materials, and also it is used to fit tree diameters data, were this data play an important role in stand model. This distribution was introduced by BAILEY and DELL (1973)(1) as a model for diameter distribution, and have been applied extensively in forestry, since it have ability to describe the wide range of Uniondale distributions. also LITTLE and KILKKI et al (1989)( 8), also Cran in (1988)( 4) work on moment estimators for three parameter weibull distribution, while Tsionas. E. G. in (2000)(13) introduced posterior analysis, prediction and reliability in three parameter weibull distribution. also weibull distribution used to describe real phenomena and modeling distribution of breaking strength of materials as indicated by Johnson et al (1994)( 7),and also murthy et al (2004)( 11). in (2000)( 13) Tsionas, work on estimating Bayesian and also prediction and reliability in three parameters weibull. and Tian. Gives inference about coefficient of variation, in (2015) ( 12)a group of researcher introduced the transmuted exponential weibull distribution with application were this distribution is flexible and useful for analyzing positive data that have bathtub shaped hazard rate function. the obtained new distribution can be used for modeling positive data in various fields like physical and biological sciences, Reliability theory hydrology, medicine and survival analysis and engineering. also in (2014) ( 5) Felix Noyanim and et al introduce comparison between different methods for estimating parameters of weibull, using mean square error as a measure for comparison. in (2013) (10), Mahdi Teimouri and Arjunk. Gupta gives more attention for estimating reliability function for three parameters weibull, and variation and maximum likelihood estimate, and they indicate to Cran (1988) ( 4), when he estimated three parameters by moments of weibull.

## Theoretical Part:

We know that Weibull distribution arises from exponential distribution, when we assume the (pth) power of failure time is random variable have an exponential distribution (i.e) if

y= $T^r$is r.v ~negative exponential with mean $\theta$ then

$$g(y,\theta) = \frac{1}{\theta}\ e^{-\frac{y}{\theta}} \qquad y,\theta > 0 \qquad \text{.....(1)}$$

Then the failure time (T) is r.v. have Weibull distribution (WEb) and its failure rate proportional to some power t, i.e

$$h(t) = \alpha\ t^k$$

$$h(t) = c\ t^k$$

$$\text{Let: } k = p - 1 \quad \text{and } c = \frac{p}{\theta}$$

Then T is r.v ~ WEb

$$f(t) = \frac{p}{\theta} \quad t^{p-1} e^{-\frac{t^p}{\theta}} p, \theta > 0 \qquad \text{..... (2)}$$

We can prove that

$\mu'_r = E(t^r)$Forf (t) in equation (1)

Equal $\mu'_r = E(t^r)$

$$= \theta^{\frac{r}{p}}\ \Gamma\ (1+\frac{r}{p}) \qquad \text{.....(3)}$$

When r=1 $\rightarrow E(T) = \theta^{\frac{1}{p}} \Gamma(1+\frac{r}{p})$

And variance of (T) is

$\sigma^2{}_T = E(T^2) + (ET)^2$

$$\sigma^2{}_T = \theta^{\frac{2}{p}}\left[\Gamma\left(1+\frac{2}{p}\right) - \Gamma^2\ (1+\frac{1}{p})\right] \quad \ldots..(4)$$

Here our research deals with three parameters Weibull which was introduced by Petrosion (1994), when the failure happensaftercertain time (i.e) $t > \delta$ , $\delta < t < \infty$:

$$f_T(t,b,c,\delta) = \frac{c}{b}\left(\frac{t-\delta}{b}\right)^{c-1} e^{-\left(\frac{t-\delta}{b}\right)^c} \quad \ldots..(5)$$

Were $\delta$: is location parameter.

b: is scale parameter.

c : shape parameter.

Corresponding to equation (5), the cumulative distribution function (CDF) is;

$$F_T(t,b,c,\delta) = 1 - e^{-\left(\frac{t-\delta}{b}\right)^c} \qquad t \geq \delta \quad \ldots..(6)$$

And the reliability function is

$R_T(t) = pr\ (T > t)$

$$R_T(t) = e^{-\left(\frac{t-\delta}{b}\right)^c} \quad \ldots..(7)$$

we also prove that the r th moments formula of three parameters weibull is;

$$\mu^x{}_r = \sum_{i=0}^{r} C_i^r \quad b^r\delta^{r-i}\Gamma\left(\frac{r}{c}+1\right) \quad \ldots..(8)$$

We can use equation (8) and Solving equation(9) ;

$$\mu^x{}_r = \frac{\sum_{i=1}^{n} ti^r}{n} \qquad \text{for } r = 1,2,3 \quad \ldots..(9)$$

to obtain $\hat{b}_{mom}$ , $\hat{c}_{mom}$ , $\hat{\delta}_{mom}$

Maximum Likelihood Method Estimation:

Let $(t_1, t_2, \ldots, t_n)$ be ar.s fromp.d.f in equation (5), then;

$$L = \prod_{i=1}^{n} f(t_i,b,c,\delta)$$

$$L = c^n\ b^{-n} \prod_{i=1}^{n} \left(\frac{t_i-\delta}{b}\right)^{c-1} e^{-\sum_{i=1}^{n}\left(\frac{t-\delta}{b}\right)^c}$$

$$L = \quad c^n \; b^{-n(c-1)} \prod_{i=1}^{n} \quad (t_i - \delta)^{c-1} \; e^{-\sum_{i=1}^{n} \left(\frac{t-\delta}{b}\right)^c} \qquad \ldots\ldots (10)$$

Taking logarithmic for equation (10) we obtain ;

$$\log L = n \log c - n\,(c-1) \log b + (c-1) \sum_{i=1}^{n} \log(t_i - \delta) - \; b^{-c} \sum_{i=1}^{n} (t_i - \delta)^c \quad \ldots (11)$$

Then from

$$\frac{\partial \log L}{\partial c} = \frac{n}{c} - n \log b + \sum_{i=1}^{n} \log(t_i - \delta) - \left( b^{-c} \sum_{i=1}^{n} (t_i - \delta)^c \,(1) \log(t_i - \delta) + \sum_{i=1}^{n} (t_i - \delta)^c \,(-c) \; b^{-c-1} \right)$$

Put $\frac{\partial \log L}{\partial c} = \quad 0$

$$\frac{n}{\hat{c}} - n \log b + \sum_{i=1}^{n} \log(t_i - \delta) - \; b^{-c} \sum_{i=1}^{n} (t_i - \delta)^c \log(t_i - \delta) - c \;\; b^{-c-1} \sum_{i=1}^{n} (t_i - \delta)^c = 0 \qquad \ldots\ldots (12)$$

Equation (12) Solved numerically to find $\hat{c}_{MLE}$

Now;

$$\frac{\partial \log L}{\partial b} = \left[ \frac{-nc}{b} + c \;\; b^{-c-1} \sum_{i=1}^{n} (t_i - \delta)^c \right\} * \quad b^{c+1} \Bigg] \div c$$

$$n * \; b^c = \sum_{i=1}^{n} (t_i - \delta)^c$$

$b^c = \frac{\sum_{i=1}^{n} (t_i - \delta)^c}{n}$ ; then

$$\hat{b}_{MLE} = \sqrt[\hat{c}]{\frac{\sum_{i=1}^{n} (t_i - \delta)^{\hat{c}}}{n}} \qquad \ldots\ldots (13)$$

And ;

$$\frac{\partial \log L}{\partial \delta} = (c-1) \sum_{i=1}^{n} \frac{-1}{(t_i - \delta)} - \; b^{-c} \sum_{i=1}^{n} c \;\; (t_i - \delta)^{c-1} \,(-1)$$

$$= (c-1) \sum_{i=1}^{n} (t_i - \delta)^{-1} + c \;\; b^{-c} \sum_{i=1}^{n} (t_i - \delta)^{c-1} \quad = 0 \qquad \ldots\ldots (14)$$

Solving equation (14) numerically to find $\left(\hat{\delta}_{MLE}\right)$

Third Method Regression Estimation:

$$F(t) = 1 - e^{-\left(\frac{t-\delta}{b}\right)^c}$$

from

$e^{-\left(\frac{t-\delta}{b}\right)^c} = 1 - F(ti)$

$$e^{-\left(\frac{t-\delta}{b}\right)^c} = wi0 \le wi \le 1 \qquad \ldots..(15)$$

Taking Logarithm for equation (15); then

$$-\left(\frac{t-\delta}{b}\right)^c = \log wi$$

we assume $\log wi = yi$ , then

$$(-yi)^{\frac{1}{c}} = \left(\frac{ti-\delta}{b}\right)$$

$$\hat{ti} = \left(b\,(-yi)^{\frac{1}{c}} + \delta\right)$$

Then estimation by regression obtainedfrom minimizing total sum of squares between; $t_i$ and $\hat{ti}$, i.e

$$T = \sum_{i=1}^{n} \left[t_{(i)} - (b\ (-1)^{\frac{1}{c}}(yi)^{\frac{1}{c}} + \delta)\right]^2$$

$$T = \sum_{i=1}^{n} \left[t_{(i)} - b\ (-yi)^{\frac{1}{c}} - \delta\right]^2 \qquad \ldots..(16)$$

From $\frac{\partial T}{\partial b} = 0$ we find

$$\hat{b}_{(Reg)} = \frac{\left(\sum_{i=1}^{n} t_{(i)}\,(-yi)^{\frac{1}{c}} - \hat{\delta}\sum_{i=1}^{n}(-yi)^{\frac{1}{c}}\right)}{\left(\sum_{i=1}^{n}(yi)^{\frac{2}{c}}\right)} \qquad \ldots..(17)$$

Also from $\frac{\partial T}{\partial c} = 0$ we obtain non linear equation, which is ;

$$\sum_{i=1}^{n}(-1)^{\frac{1}{c}}\ t_{(i)}yi\ \log(\frac{1}{yi}) - b\sum_{i=1}^{n}(-yi)^{\frac{2}{c}}\log(\frac{1}{yi}) + \hat{\delta}\sum_{i=1}^{n}(-1)^{\frac{1}{c}}\ yi\ \log(yi^{-1}) = 0 \qquad \ldots..(18)$$

from equation (18) solved numerically To find $\hat{c}_{(Reg)}$

$$\frac{\partial T}{\partial \delta} = 2\sum_{i=1}^{n}\ (t_{(i)} - b - (yi)^{\frac{1}{c}} - \delta)\ (1) = 0$$

$$= \sum_{i=1}^{n}\ t_{(i)} - n\hat{b} - \sum_{i=1}^{n}(yi)^{\frac{1}{c}} = n\hat{\delta} \qquad \div n$$

$$\hat{\delta}_{MLE} = t - \hat{b} - \frac{\sum_{i=1}^{n}(yi)^{\frac{1}{c}}}{n} \qquad \ldots\ldots(19)$$

Simulation Procedure:

To find $(\hat{b}, \hat{c}, \hat{\delta})$ we apply simulation procedure, taking sample size $(n = 40,60,80)$, and using inverse transformation of (CDF) as ;

$$U_i = F(t_i, \quad \delta, b, \quad c)$$

$$U_i = 1 - e^{-\left(\frac{t_i-\delta}{b}\right)^c} \qquad t \le \delta$$

$$e^{-\left(\frac{t_i-\delta}{b}\right)^c} = 1 - U_i$$

$$\log(1 - U_i) = -\left(\frac{t_i - \delta}{b}\right)^c \qquad 0 \le U_i \le 1$$

$$-\log(1 - U_i) = \left(\frac{t_i - \delta}{b}\right)^c$$

Let $zi = -\log(1 - U_i)$ and $wi = (zi)^{\frac{1}{c}}$

Then $zi = \left(\frac{t_i - \delta}{b}\right)^c$

when $wi = (zi)^{\frac{1}{c}}$

Then $(zi)^{\frac{1}{c}} = \frac{t_i-\delta}{b}$

$$wi = \frac{t_i - \delta}{b}$$

$$b\,wi = t_i - \delta$$

$$t_i = b\,wi + \delta \qquad t_i \ge \delta$$

Using generated values $(t_i)$ for (n =40, 60, 80) we can find Moment (estimators) from equation (9) and maximum likelihood from equations( 12,13,14), and finally we apply Regression analysis of $(t_i)$ ( at n= 40, 60, 80 ) to obtain least square estimator for $(\hat{b}, \hat{c}, \hat{\delta})$; from equation (17,18,19)

and the Results of estimation are compared using statistical measure mean square error (MSE), and we explained in the following table we notice some values of $\hat{h}(t_i)$, are greater than (1), this indicates that the value of p.d.f at some set is greater that the value of $R_T(t)$, at this set.

Table (1)

contains table (1.1), table (1.2), table(1.3),and table(1.4)

Table (1.1) estimator fuzzy hazard rate when ( c=3, b=2.8, ʒ=0.5, ǩ =0.3)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.1584 | 0.0994 | 0.0151 | 0.0379 | 0.0007 | 0.0041 | 0.0045 | mom |
| | 1.1500 | 0.2548 | 0.0999 | 0.0091 | 0.0379 | 0.0048 | 0.0121 | 0.0122 | mom |
| | 1.4500 | 0.3513 | 0.1004 | 0.0332 | 0.1137 | 0.0126 | 0.0202 | 0.0215 | mom |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1.7500 | 0.4477 | 0.1009 | 0.0574 | 0.1895 | 0.0241 | 0.0305 | 0.0334 | mom |
| | 2.0500 | 0.5441 | 0.1014 | 0.0816 | 0.2653 | 0.0392 | 0.0428 | 0.0478 | mom |
| 60 | 0.8500 | 0.1584 | 0.1019 | 0.0175 | 0.0202 | 0.0006 | 0.0040 | 0.0038 | mom |
| | 1.1500 | 0.2548 | 0.1024 | 0.0094 | 0.0202 | 0.0046 | 0.0120 | 0.0110 | mom |
| | 1.4500 | 0.3513 | 0.1029 | 0.0363 | 0.0606 | 0.0123 | 0.0198 | 0.0169 | mom |
| | 1.7500 | 0.4477 | 0.1034 | 0.0632 | 0.1010 | 0.0237 | 0.0296 | 0.0240 | mom |
| | 2.0500 | 0.5441 | 0.1040 | 0.0901 | 0.1415 | 0.0387 | 0.0412 | 0.0324 | reg |
| 80 | 0.8500 | 0.1584 | 0.1347 | 0.0422 | 0.0079 | 0.0001 | 0.0027 | 0.0029 | mom |
| | 1.1500 | 0.2548 | 0.1357 | 0.0001 | 0.0079 | 0.0028 | 0.0130 | 0.0094 | mom |
| | 1.4500 | 0.3513 | 0.1366 | 0.0419 | 0.0236 | 0.0092 | 0.0191 | 0.0113 | mom |
| | 1.7500 | 0.4477 | 0.1376 | 0.0840 | 0.0393 | 0.0192 | 0.0265 | 0.0133 | reg |
| | 2.0500 | 0.5441 | 0.1386 | 0.1260 | 0.0551 | 0.0329 | 0.0350 | 0.0156 | reg |

Table (1.2) estimator fuzzy hazard rate when ( c=3, b=2.8, $\breve{z}$=0.5, $\breve{k}$ =0.6)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.3551 | 0.0878 | 0.0005 | 0.0069 | 0.0143 | 0.0251 | 0.0251 | mom |
| | 1.1500 | 0.5480 | 0.0886 | 0.0109 | 0.1315 | 0.0422 | 0.0577 | 0.0568 | mom |
| | 1.4500 | 0.7408 | 0.0893 | 0.0224 | 0.2562 | 0.0849 | 0.1032 | 0.1012 | mom |
| | 1.7500 | 0.9337 | 0.0901 | 0.0338 | 0.3808 | 0.1423 | 0.1619 | 0.1585 | mom |
| | 2.0500 | 1.1265 | 0.0909 | 0.0453 | 0.5054 | 0.2145 | 0.2338 | 0.2284 | mom |
| 60 | 0.8500 | 0.3551 | 0.1209 | 0.0002 | 0.0008 | 0.0110 | 0.0251 | 0.0250 | mom |
| | 1.1500 | 0.5480 | 0.1224 | 0.0446 | 0.0151 | 0.0362 | 0.0507 | 0.0545 | mom |
| | 1.4500 | 0.7408 | 0.1239 | 0.0894 | 0.0293 | 0.0761 | 0.0849 | 0.0953 | mom |
| | 1.7500 | 0.9337 | 0.1254 | 0.1342 | 0.0436 | 0.1307 | 0.1278 | 0.1474 | Mle |
| | 2.0500 | 1.1265 | 0.1269 | 0.1790 | 0.0578 | 0.1998 | 0.1795 | 0.2108 | Mle |
| | 0.8500 | 0.3551 | 0.1491 | 0.0018 | 0.0014 | 0.0085 | 0.0250 | 0.0242 | mom |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 80 | 1.1500 | 0.5480 | 0.1514 | 0.0485 | 0.0260 | 0.0314 | 0.0499 | 0.0347 | mom |
| | 1.4500 | 0.7408 | 0.1538 | 0.0988 | 0.0506 | 0.0689 | 0.0824 | 0.0470 | reg |
| | 1.7500 | 0.9337 | 0.1561 | 0.1491 | 0.0752 | 0.1209 | 0.1231 | 0.0611 | reg |
| | 2.0500 | 1.1265 | 0.1585 | 0.1994 | 0.0999 | 0.1874 | 0.1719 | 0.0772 | reg |

Table (1.3) estimator fuzzy hazard rate when ( c=3, b=3.2, ʒ=0.5, ǩ =0.3)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.0926 | 0.0892 | 0.0991 | 0.0669 | 0.0002 | 0.0001 | 0.0004 | mle |
| | 1.1500 | 0.1770 | 0.0896 | 0.0647 | 0.0423 | 0.0015 | 0.0025 | 0.0036 | mom |
| | 1.4500 | 0.2613 | 0.0900 | 0.0303 | 0.0178 | 0.0107 | 0.0059 | 0.0119 | mle |
| | 1.7500 | 0.3457 | 0.0904 | 0.0041 | 0.0068 | 0.0230 | 0.0233 | 0.0130 | reg |
| | 2.0500 | 0.4301 | 0.0908 | 0.0385 | 0.0314 | 0.0307 | 0.0230 | 0.0318 | mle |
| 60 | 0.8500 | 0.0926 | 0.0909 | 0.0581 | 0.0945 | 0.0001 | 0.0002 | 0.0003 | mom |
| | 1.1500 | 0.1770 | 0.0913 | 0.0383 | 0.0598 | 0.0015 | 0.0023 | 0.0027 | mom |
| | 1.4500 | 0.2613 | 0.0918 | 0.0184 | 0.0251 | 0.0102 | 0.0058 | 0.0112 | mle |
| | 1.7500 | 0.3457 | 0.0922 | 0.0014 | 0.0096 | 0.0231 | 0.0129 | 0.0226 | mle |
| | 2.0500 | 0.4301 | 0.0926 | 0.0213 | 0.0443 | 0.0304 | 0.0228 | 0.0298 | mle |
| 80 | 0.8500 | 0.0926 | 0.1092 | 0.0529 | 0.0781 | 0.0001 | 0.0003 | 0.0001 | mom |
| | 1.1500 | 0.1770 | 0.1099 | 0.0345 | 0.0494 | 0.0023 | 0.0041 | 0.0009 | reg |
| | 1.4500 | 0.2613 | 0.1105 | 0.0161 | 0.0207 | 0.0120 | 0.0045 | 0.0107 | mle |
| | 1.7500 | 0.3457 | 0.1112 | 0.0023 | 0.0080 | 0.0110 | 0.0236 | 0.0220 | mom |
| | 2.0500 | 0.4301 | 0.1118 | 0.0207 | 0.0367 | 0.0287 | 0.0335 | 0.0203 | reg |

Table (1.4) estimator fuzzy hazard rate when ( c=3, b=3.2, ʒ=0.5, ǩ =0.6)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| | 0.8500 | 0.3316 | 0.0857 | 0.0009 | 0.0023 | 0.0121 | 0.0219 | 0.0219 | mom |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 1.1500 | 0.5004 | 0.0865 | 0.0615 | 0.0443 | 0.0343 | 0.0385 | 0.0470 | mom |
| | 1.4500 | 0.6691 | 0.0872 | 0.1240 | 0.0863 | 0.0677 | 0.0594 | 0.0816 | mle |
| | 1.7500 | 0.8379 | 0.0880 | 0.1865 | 0.1284 | 0.1125 | 0.0849 | 0.1257 | mle |
| | 2.0500 | 1.0066 | 0.0887 | 0.2489 | 0.1704 | 0.1685 | 0.1793 | 0.1148 | reg |
| 60 | 0.8500 | 0.3316 | 0.1106 | 0.0018 | 0.0008 | 0.0098 | 0.0218 | 0.0217 | mom |
| | 1.1500 | 0.5004 | 0.1119 | 0.1755 | 0.0156 | 0.0302 | 0.0211 | 0.0416 | mle |
| | 1.4500 | 0.6691 | 0.1132 | 0.3492 | 0.0303 | 0.0618 | 0.0679 | 0.0205 | reg |
| | 1.7500 | 0.8379 | 0.1145 | 0.5228 | 0.0450 | 0.1047 | 0.0199 | 0.1007 | mle |
| | 2.0500 | 1.0066 | 0.1157 | 0.6965 | 0.0598 | 0.1587 | 0.0192 | 0.1399 | mle |
| 80 | 0.8500 | 0.3316 | 0.1266 | 0.0009 | 0.0009 | 0.0084 | 0.0215 | 0.0215 | mom |
| | 1.1500 | 0.5004 | 0.1283 | 0.0172 | 0.0177 | 0.0277 | 0.0167 | 0.0410 | mle |
| | 1.4500 | 0.6691 | 0.1300 | 0.0353 | 0.0345 | 0.0581 | 0.0543 | 0.0204 | reg |
| | 1.7500 | 0.8379 | 0.1317 | 0.0534 | 0.0513 | 0.0997 | 0.0183 | 0.1007 | mle |
| | 2.0500 | 1.0066 | 0.1334 | 0.0715 | 0.0681 | 0.1525 | 0.0149 | 0.1262 | mle |

Table (2)

contains table (2.1), table (2.2), table(2.3),and table(2.4)

Table (2.1) estimator fuzzy hazard rate when ( c=5, b=2.8, ʒ=0.5, ǩ =0.3)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.1543 | 0.1414 | 0.0083 | 0.0017 | 0.0043 | 0.0001 | 0.0047 | mle |
| | 1.1500 | 0.2949 | 0.1424 | 0.0054 | 0.0011 | 0.0168 | 0.0047 | 0.0173 | mle |
| | 1.4500 | 0.4355 | 0.1435 | 0.0025 | 0.0005 | 0.0171 | 0.0375 | 0.0379 | mom |
| | 1.7500 | 0.5762 | 0.1445 | 0.0004 | 0.0002 | 0.0664 | 0.0663 | 0.0373 | reg |
| | 2.0500 | 0.7168 | 0.1456 | 0.0033 | 0.0008 | 0.0653 | 0.1018 | 0.1027 | mom |
| 60 | 0.8500 | 0.1543 | 0.1777 | 0.0115 | 0.7475e-3 | 0.0001 | 0.0041 | 0.0047 | mom |
| | 1.1500 | 0.2949 | 0.1794 | 0.0075 | 0.4729e-4 | 0.0165 | 0.0027 | 0.0170 | mle |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1.4500 | 0.4355 | 0.1812 | 0.0035 | 0.1983e-3 | 0.0373 | 0.0129 | 0.0379 | mle |
| | 1.7500 | 0.5762 | 0.1829 | 0.0005 | 0.0763e-3 | 0.0309 | 0.0663 | 0.0660 | mom |
| | 2.0500 | 0.7168 | 0.1846 | 0.0045 | 0.3509e-3 | 0.1015 | 0.0566 | 0.1025 | mle |
| 80 | 0.8500 | 0.1543 | 0.2129 | 0.0093 | 0.2285e-3 | 0.0047 | 0.0040 | 0.0001 | reg |
| | 1.1500 | 0.2949 | 0.2155 | 0.0062 | 0.1446e-3 | 0.0013 | 0.0162 | 0.0167 | mom |
| | 1.4500 | 0.4355 | 0.2180 | 0.0030 | 0.0606e-3 | 0.0370 | 0.0095 | 0.0379 | mle |
| | 1.7500 | 0.5762 | 0.2205 | 0.0001 | 0.0233e-3 | 0.0253 | 0.0662 | 0.0658 | mom |
| | 2.0500 | 0.7168 | 0.2230 | 0.0033 | 0.1073e-3 | 0.1012 | 0.0488 | 0.1025 | mle |

Table (2.2) estimator fuzzy hazard rate when ( c=5, b=2.8, ʒ=0.5, ǩ =0.6)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.5527 | 0.1383 | 0.0001 | 0.0002 | 0.0344 | 0.0611 | 0.0611 | Mom |
| | 1.1500 | 0.8340 | 0.1402 | 0.0061 | 0.0040 | 0.1371 | 0.0963 | 0.1378 | Mle |
| | 1.4500 | 1.1152 | 0.1422 | 0.0123 | 0.0078 | 0.2433 | 0.1894 | 0.2453 | mle |
| | 1.7500 | 1.3965 | 0.1441 | 0.0185 | 0.0117 | 0.3137 | 0.3798 | 0.3835 | Mom |
| | 2.0500 | 1.6777 | 0.1460 | 0.0247 | 0.0155 | 0.5526 | 0.5465 | 0.4692 | reg |
| 60 | 0.8500 | 0.5527 | 0.1861 | 0.0002 | 0.0100 | 0.0611 | 0.0269 | 0.0611 | Mle |
| | 1.1500 | 0.8340 | 0.1896 | 0.0066 | 0.1895 | 0.1369 | 0.0830 | 0.1362 | Mle |
| | 1.4500 | 1.1152 | 0.1932 | 0.0131 | 0.3690 | 0.1700 | 0.2429 | 0.2444 | Mom |
| | 1.7500 | 1.3965 | 0.1968 | 0.0196 | 0.5485 | 0.3792 | 0.2878 | 0.3833 | Mle |
| | 2.0500 | 1.6777 | 0.2004 | 0.0260 | 0.7280 | 0.4465 | 0.5456 | 0.5525 | Mom |
| 80 | 0.8500 | 0.5527 | 0.1823 | 0.0002 | 0.0000 | 0.0610 | 0.0263 | 0.0611 | Mle |
| | 1.1500 | 0.8340 | 0.1858 | 0.0059 | 0.0009 | 0.0822 | 0.1354 | 0.1360 | Mom |
| | 1.4500 | 1.1152 | 0.1893 | 0.0121 | 0.0018 | 0.2431 | 0.1687 | 0.2434 | Mle |
| | 1.7500 | 1.3965 | 0.1928 | 0.0183 | 0.0027 | 0.3832 | 0.3790 | 0.2866 | reg |
| | 2.0500 | 1.6777 | 0.1963 | 0.0245 | 0.0035 | 0.5451 | 0.4390 | 0.5506 | Mle |

Table (2.3) estimator fuzzy hazard rate when ( c=5, b=3.2, ʒ=0.5, ǩ =0.3)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.1543 | 0.1414 | 0.0083 | 0.0017 | 0.0047 | 0.0043 | 0.0001 | reg |
| | 1.1500 | 0.2949 | 0.1424 | 0.0054 | 0.0011 | 0.0047 | 0.0168 | 0.0173 | Mom |
| | 1.4500 | 0.4355 | 0.1435 | 0.0025 | 0.0005 | 0.0375 | 0.0171 | 0.0379 | Mle |
| | 1.7500 | 0.5762 | 0.1445 | 0.0004 | 0.0002 | 0.0373 | 0.0663 | 0.0664 | Mom |
| | 2.0500 | 0.7168 | 0.1456 | 0.0033 | 0.0008 | 0.1018 | 0.0653 | 0.1025 | Mle |
| 60 | 0.8500 | 0.1543 | 0.1777 | 0.0115 | 0.7475e-3 | 0.0041 | 0.0001 | 0.0045 | Mle |
| | 1.1500 | 0.2949 | 0.1794 | 0.0075 | 0.4729e-3 | 0.0165 | 0.0027 | 0.0170 | Mle |
| | 1.4500 | 0.4355 | 0.1812 | 0.0035 | 0.1983e-3 | 0.0373 | 0.0129 | 0.0375 | Mle |
| | 1.7500 | 0.5762 | 0.1829 | 0.0005 | 0.0763e-3 | 0.0309 | 0.0663 | 0.0664 | Mom |
| | 2.0500 | 0.7168 | 0.1846 | 0.0045 | 0.3509e-3 | 0.1020 | 0.1015 | 0.0566 | reg |
| 80 | 0.8500 | 0.1543 | 0.1881 | 0.0074 | 0.5662e-3 | 0.0002 | 0.0038 | 0.0042 | Mom |
| | 1.1500 | 0.2949 | 0.1900 | 0.0049 | 0.3582e-3 | 0.0022 | 0.0162 | 0.0165 | Mom |
| | 1.4500 | 0.4355 | 0.1919 | 0.0023 | 0.1502e-3 | 0.0373 | 0.0119 | 0.0371 | Mle |
| | 1.7500 | 0.5762 | 0.1938 | 0.0002 | 0.0578e-3 | 0.0292 | 0.0661 | 0.0664 | Mom |
| | 2.0500 | 0.7168 | 0.1957 | 0.0027 | 0.2658e-3 | 0.1010 | 0.0543 | 0.1018 | Mle |

Table (2.4) estimator fuzzy hazard rate when ( c=5, b=3.2, ʒ=0.5, ǩ =0.6)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.5527 | 0.1392 | 0.0001 | 0.0002 | 0.0342 | 0.0611 | 0.0611 | Mom |
| | 1.1500 | 0.8340 | 0.1412 | 0.0085 | 0.0040 | 0.0960 | 0.1363 | 0.1391 | Mom |
| | 1.4500 | 1.1152 | 0.1432 | 0.0172 | 0.0077 | 0.2411 | 0.1890 | 0.2486 | Mle |
| | 1.7500 | 1.3965 | 0.1452 | 0.0259 | 0.0115 | 0.3757 | 0.3132 | 0.3898 | Mle |
| | 2.0500 | 1.6777 | 0.1471 | 0.0346 | 0.0152 | 0.5400 | 0.4686 | 0.5528 | Mle |
| | 0.8500 | 0.5527 | 0.1866 | 0.0002 | 0.0076 | 0.0611 | 0.0610 | 0.0268 | reg |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 60 | 1.1500 | 0.8340 | 0.1902 | 0.0071 | 0.1444 | 0.0829 | 0.1361 | 0.1378 | Mom |
| | 1.4500 | 1.1152 | 0.1937 | 0.0139 | 0.2812 | 0.2408 | 0.1698 | 0.2453 | Mle |
| | 1.7500 | 1.3965 | 0.1973 | 0.0207 | 0.4180 | 0.3753 | 0.2876 | 0.3832 | Mle |
| | 2.0500 | 1.6777 | 0.2009 | 0.0275 | 0.5548 | 0.4362 | 0.5322 | 0.5526 | Mom |
| 80 | 0.8500 | 0.5527 | 0.1889 | 0.0006 | 0.0000 | 0.0610 | 0.0265 | 0.0610 | Mle |
| | 1.1500 | 0.8340 | 0.1926 | 0.0103 | 0.0003 | 0.1357 | 0.0823 | 0.1365 | Mle |
| | 1.4500 | 1.1152 | 0.1964 | 0.0211 | 0.0006 | 0.1689 | 0.2394 | 0.2451 | Mom |
| | 1.7500 | 1.3965 | 0.2002 | 0.0320 | 0.0009 | 0.3830 | 0.3724 | 0.2862 | reg |
| | 2.0500 | 1.6777 | 0.2039 | 0.0428 | 0.0012 | 0.4344 | 0.5312 | 0.5521 | Mom |

Table (3)

contains table (3.1), table (3.2), table(3.3),and table(3.4)

Table (3.1) estimator fuzzy hazard rate when ( c=5, b=2.8, ʒ=0.8, ǩ =0.3)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.1365 | 0.1611 | 0.0343 | 0.0016 | 0.0020 | 0.0021 | 0.0037 | Mom |
| | 1.1500 | 0.2972 | 0.1625 | 0.0222 | 0.0010 | 0.0176 | 0.0158 | 0.0036 | reg |
| | 1.4500 | 0.4579 | 0.1638 | 0.0102 | 0.0004 | 0.0401 | 0.0173 | 0.0419 | Mle |
| | 1.7500 | 0.6186 | 0.1652 | 0.0019 | 0.0002 | 0.0411 | 0.0761 | 0.0768 | Mom |
| | 2.0500 | 0.7793 | 0.1666 | 0.0139 | 0.0007 | 0.1172 | 0.0751 | 0.1212 | Mle |
| 60 | 0.8500 | 0.1365 | 0.2023 | 0.0476 | 0.7108 | 0.0025 | 0.0009 | 0.0036 | Mle |
| | 1.1500 | 0.2972 | 0.2046 | 0.0309 | 0.4497 | 0.0017 | 0.0151 | 0.0173 | Mom |
| | 1.4500 | 0.4579 | 0.2068 | 0.0143 | 0.1886 | 0.0419 | 0.0394 | 0.0126 | reg |
| | 1.7500 | 0.6186 | 0.2091 | 0.0023 | 0.0725 | 0.0760 | 0.0335 | 0.0765 | Mle |
| | 2.0500 | 0.7793 | 0.2113 | 0.0190 | 0.3336 | 0.1156 | 0.0645 | 0.1210 | Mle |
| | 0.8500 | 0.1365 | 0.2423 | 0.0239 | 0.2054 | 0.0001 | 0.0016 | 0.0036 | Mom |
| | 1.1500 | 0.2972 | 0.2455 | 0.0158 | 0.1299 | 0.0005 | 0.0142 | 0.0170 | Mom |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 80 | 1.4500 | 0.4579 | 0.2488 | 0.0076 | 0.0545 | 0.0388 | 0.0087 | 0.0419 | Mle |
| | 1.7500 | 0.6186 | 0.2521 | 0.0006 | 0.0210 | 0.0763 | 0.0758 | 0.0269 | reg |
| | 2.0500 | 0.7793 | 0.2554 | 0.0088 | 0.0964 | 0.1204 | 0.1152 | 0.0549 | Mle |

Table (3.2) estimator fuzzy hazard rate when ( c=5, b=2.8, ʒ=0.8, ǩ =0.6)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.4005 | 0.1539 | 0.2330 | 0.0030 | 0.0122 | 0.0056 | 0.0321 | Mle |
| | 1.1500 | 0.7219 | 0.1564 | 0.0932 | 0.0011 | 0.0640 | 0.0791 | 0.1042 | Mom |
| | 1.4500 | 1.0434 | 0.1589 | 0.0466 | 0.0007 | 0.1565 | 0.1987 | 0.2177 | Mom |
| | 1.7500 | 1.3648 | 0.1614 | 0.1864 | 0.0026 | 0.2896 | 0.2777 | 0.3725 | Mle |
| | 2.0500 | 1.6862 | 0.1640 | 0.3262 | 0.0045 | 0.4635 | 0.3699 | 0.5685 | Mle |
| 60 | 0.8500 | 0.4005 | 0.2049 | 0.2755 | 0.1314 | 0.0077 | 0.0031 | 0.0316 | Mle |
| | 1.1500 | 0.7219 | 0.2096 | 0.1064 | 0.0499 | 0.0525 | 0.0758 | 0.1039 | Mom |
| | 1.4500 | 1.0434 | 0.2142 | 0.0628 | 0.0317 | 0.1375 | 0.1923 | 0.2174 | Mom |
| | 1.7500 | 1.3648 | 0.2189 | 0.2319 | 0.1133 | 0.2626 | 0.2567 | 0.3711 | Mle |
| | 2.0500 | 1.6862 | 0.2236 | 0.4011 | 0.1949 | 0.4279 | 0.5656 | 0.3303 | reg |
| 80 | 0.8500 | 0.4005 | 0.2008 | 0.4318 | 0.6643 | 0.0080 | 0.0002 | 0.0314 | Mle |
| | 1.1500 | 0.7219 | 0.2053 | 0.1770 | 0.2520 | 0.1035 | 0.0594 | 0.0534 | reg |
| | 1.4500 | 1.0434 | 0.2099 | 0.0778 | 0.1603 | 0.1389 | 0.1865 | 0.2172 | Mom |
| | 1.7500 | 1.3648 | 0.2145 | 0.3326 | 0.5727 | 0.2647 | 0.2131 | 0.3708 | Mle |
| | 2.0500 | 1.6862 | 0.2190 | 0.5874 | 0.9850 | 0.4305 | 0.2415 | 0.5648 | Mle |

Table (3.3) estimator fuzzy hazard rate when ( c=5, b=3.2, ʒ=0.8, ǩ =0.3)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.0078 | 0.1272 | 0.0406 | 0.0053 | 0.0068 | 0.0004 | 0.0001 | Reg |
| | 1.1500 | 0.1484 | 0.1281 | 0.0341 | 0.0044 | 0.0026 | 0.0004 | 0.0044 | Mle |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1.4500 | 0.2891 | 0.1290 | 0.0275 | 0.0036 | 0.0137 | 0.0051 | 0.0167 | Mle |
| | 1.7500 | 0.4297 | 0.1299 | 0.0209 | 0.0027 | 0.0180 | 0.0334 | 0.0369 | Mom |
| | 2.0500 | 0.5703 | 0.1308 | 0.0143 | 0.0018 | 0.0386 | 0.0618 | 0.0650 | Mom |
| 60 | 0.8500 | 0.0078 | 0.1773 | 0.0542 | 0.3271 | 0.0057 | 0.0002 | 0.0001 | Reg |
| | 1.1500 | 0.1484 | 0.1791 | 0.0455 | 0.2731 | 0.0002 | 0.0021 | 0.0041 | Mom |
| | 1.4500 | 0.2891 | 0.1810 | 0.0367 | 0.2191 | 0.0127 | 0.0023 | 0.0163 | Mle |
| | 1.7500 | 0.4297 | 0.1828 | 0.0280 | 0.1651 | 0.0122 | 0.0323 | 0.0365 | Mom |
| | 2.0500 | 0.5703 | 0.1846 | 0.0192 | 0.1110 | 0.0607 | 0.0297 | 0.0649 | Mle |
| 80 | 0.8500 | 0.0078 | 0.1928 | 0.0697 | 0.7447 | 0.0023 | 0.0002 | 0.0001 | Reg |
| | 1.1500 | 0.1484 | 0.1951 | 0.0584 | 0.6217 | 0.0001 | 0.0016 | 0.0040 | Mom |
| | 1.4500 | 0.2891 | 0.1974 | 0.0471 | 0.4987 | 0.0117 | 0.0017 | 0.0160 | Mle |
| | 1.7500 | 0.4297 | 0.1997 | 0.0358 | 0.3758 | 0.0106 | 0.0310 | 0.0362 | Mom |
| | 2.0500 | 0.5703 | 0.2020 | 0.0245 | 0.2528 | 0.0596 | 0.0271 | 0.0645 | Mle |

Table (3.4) estimator fuzzy hazard rate when ( c=5, b=3.2, ʒ=0.8, ǩ =0.6)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| | 0.8500 | 0.4063 | 0.1302 | 0.0109 | 0.0058 | 0.0313 | 0.0152 | 0.0391 | Mle |
| | 1.1500 | 0.6875 | 0.1319 | 0.0043 | 0.0022 | 0.0617 | 0.0934 | 0.0945 | Mom |
| 40 | 1.4500 | 0.9688 | 0.1337 | 0.0024 | 0.0014 | 0.1877 | 0.1868 | 0.1395 | reg |
| | 1.7500 | 1.2500 | 0.1355 | 0.0090 | 0.0050 | 0.3100 | 0.3080 | 0.2484 | reg |
| | 2.0500 | 1.5312 | 0.1373 | 0.0156 | 0.0086 | 0.3886 | 0.4594 | 0.4688 | Mom |
| | 0.8500 | 0.4063 | 0.1625 | 0.0546 | 0.1817 | 0.0119 | 0.0247 | 0.0386 | Mom |
| | 1.1500 | 0.6875 | 0.1654 | 0.0220 | 0.0689 | 0.0886 | 0.0545 | 0.0939 | Mle |
| 60 | 1.4500 | 0.9688 | 0.1683 | 0.0105 | 0.0439 | 0.1836 | 0.1281 | 0.1872 | Mle |
| | 1.7500 | 1.2500 | 0.1712 | 0.0431 | 0.1566 | 0.2328 | 0.2913 | 0.3097 | Mom |
| | 2.0500 | 1.5312 | 0.1741 | 0.0756 | 0.2694 | 0.4238 | 0.3684 | 0.4637 | Mle |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 80 | 0.8500 | 0.4063 | 0.1952 | 0.1952 | 0.0249 | 0.0110 | 0.0089 | 0.0295 | Mle |
| | 1.1500 | 0.6875 | 0.1995 | 0.1995 | 0.0106 | 0.0538 | 0.0476 | 0.0917 | Mle |
| | 1.4500 | 0.9688 | 0.2038 | 0.2038 | 0.0038 | 0.0442 | 0.1170 | 0.1862 | Mom |
| | 1.7500 | 1.2500 | 0.2082 | 0.2082 | 0.0182 | 0.3035 | 0.2171 | 0.1579 | reg |
| | 2.0500 | 1.5312 | 0.2125 | 0.2125 | 0.0326 | 0.2716 | 0.3478 | 0.4292 | Mom |

Table (4)

contains table (4.1), table (4.2), table(4.3),and table(4.4)

Table (4.1) estimator fuzzy hazard rate when ( c=3, b=2.8, ʒ=0.8, ǩ =0.3)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.0329 | 0.1002 | 0.4426 | 0.1682 | 0.0055 | 0.0945 | 0.0081 | Mom |
| | 1.1500 | 0.0635 | 0.1007 | 0.3708 | 0.1404 | 0.0010 | 0.0589 | 0.0012 | Mom |
| | 1.4500 | 0.1599 | 0.1013 | 0.2989 | 0.1127 | 0.0007 | 0.0139 | 0.0004 | Reg |
| | 1.7500 | 0.2564 | 0.1018 | 0.2271 | 0.0849 | 0.0048 | 0.0020 | 0.0059 | Mom |
| | 2.0500 | 0.3528 | 0.1023 | 0.1553 | 0.0571 | 0.0125 | 0.0078 | 0.0275 | Mle |
| 60 | 0.8500 | 0.0329 | 0.1134 | 0.6543 | 0.0919 | 0.0043 | 0.0423 | 0.0031 | Reg |
| | 1.1500 | 0.0635 | 0.1142 | 0.5501 | 0.0768 | 0.0005 | 0.0474 | 0.0001 | Reg |
| | 1.4500 | 0.1599 | 0.1149 | 0.4459 | 0.0616 | 0.0004 | 0.0133 | 0.0019 | Mom |
| | 1.7500 | 0.2564 | 0.1156 | 0.3416 | 0.0464 | 0.0040 | 0.0015 | 0.0045 | Mle |
| | 2.0500 | 0.3528 | 0.1163 | 0.2374 | 0.0312 | 0.0112 | 0.0027 | 0.0227 | Mle |
| 80 | 0.8500 | 0.0329 | 0.1337 | 0.4942 | 0.0487 | 0.0035 | 0.0356 | 0.0013 | Reg |
| | 1.1500 | 0.0635 | 0.1347 | 0.4157 | 0.0406 | 0.0003 | 0.0248 | 0.0001 | Mom |
| | 1.4500 | 0.1599 | 0.1358 | 0.3373 | 0.0326 | 0.0001 | 0.0123 | 0.0002 | Mom |
| | 1.7500 | 0.2564 | 0.1368 | 0.2588 | 0.0246 | 0.0029 | 0.0010 | 0.0032 | Mle |
| | 2.0500 | 0.3528 | 0.1379 | 0.1804 | 0.0165 | 0.0092 | 0.0017 | 0.0206 | Mle |

Table (4.2) estimator fuzzy hazard rate when ( c=3, b=2.8, ʒ=0.8, ǩ =0.6)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.2403 | 0.1454 | 0.1952 | 0.0296 | 0.0033 | 0.0026 | 0.0089 | Mle |
| | 1.1500 | 0.4332 | 0.1478 | 0.0891 | 0.0112 | 0.0205 | 0.0237 | 0.0356 | Mom |
| | 1.4500 | 0.6260 | 0.1502 | 0.0169 | 0.0072 | 0.0553 | 0.0742 | 0.0766 | Mom |
| | 1.7500 | 0.8189 | 0.1526 | 0.1230 | 0.0255 | 0.0988 | 0.0969 | 0.1259 | Mle |
| | 2.0500 | 1.0117 | 0.1550 | 0.2290 | 0.0439 | 0.1468 | 0.1225 | 0.1873 | Mle |
| 60 | 0.8500 | 0.2403 | 0.1115 | 0.3508 | 0.0302 | 0.0024 | 0.0022 | 0.0088 | Mle |
| | 1.1500 | 0.4332 | 0.1128 | 0.1417 | 0.0114 | 0.0163 | 0.0170 | 0.0356 | Mom |
| | 1.4500 | 0.6260 | 0.1141 | 0.0675 | 0.0073 | 0.0524 | 0.0634 | 0.0761 | Mom |
| | 1.7500 | 0.8189 | 0.1155 | 0.2767 | 0.0260 | 0.0981 | 0.0588 | 0.1257 | Mle |
| | 2.0500 | 1.0117 | 0.1168 | 0.4858 | 0.0447 | 0.1402 | 0.0553 | 0.1870 | Mle |
| 80 | 0.8500 | 0.2403 | 0.1300 | 0.1263 | 0.0214 | 0.0018 | 0.0004 | 0.0078 | Mle |
| | 1.1500 | 0.4332 | 0.1319 | 0.0521 | 0.0081 | 0.0157 | 0.0167 | 0.0351 | Mom |
| | 1.4500 | 0.6260 | 0.1338 | 0.0222 | 0.0052 | 0.0485 | 0.0629 | 0.0756 | Mom |
| | 1.7500 | 0.8189 | 0.1356 | 0.0965 | 0.0185 | 0.0934 | 0.0444 | 0.1243 | Mle |
| | 2.0500 | 1.0117 | 0.1375 | 0.1708 | 0.0318 | 0.1399 | 0.0414 | 0.1796 | Mle |

Table (4.3) estimator fuzzy hazard rate when ( c=3, b=3.2, ʒ=0.8, ǩ =0.3)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.0047 | 0.0812 | 0.4008 | 0.1008 | 0.0025 | 0.0614 | 0.0018 | Reg |
| | 1.1500 | 0.0891 | 0.0816 | 0.3364 | 0.0842 | 0.0001 | 0.0164 | 0.0010 | Mom |
| | 1.4500 | 0.1734 | 0.0819 | 0.2720 | 0.0675 | 0.0017 | 0.0077 | 0.0054 | Mom |
| | 1.7500 | 0.2578 | 0.0823 | 0.2076 | 0.0509 | 0.0062 | 0.0005 | 0.0086 | Mle |
| | 2.0500 | 0.3422 | 0.0826 | 0.1432 | 0.0342 | 0.0135 | 0.0079 | 0.0190 | Mle |
| 60 | 0.8500 | 0.0047 | 0.1038 | 0.5352 | 0.0244 | 0.0020 | 0.0563 | 0.0008 | Reg |
| | 1.1500 | 0.0891 | 0.1044 | 0.4491 | 0.0204 | 0.0001 | 0.0159 | 0.0009 | Mom |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1.4500 | 0.1734 | 0.1050 | 0.3631 | 0.0164 | 0.0009 | 0.0072 | 0.0049 | Mom |
| | 1.7500 | 0.2578 | 0.1056 | 0.2770 | 0.0123 | 0.0046 | 0.0003 | 0.0021 | Mle |
| | 2.0500 | 0.3422 | 0.1062 | 0.1909 | 0.0083 | 0.0111 | 0.0046 | 0.0123 | Mle |
| 80 | 0.8500 | 0.0047 | 0.1088 | 0.0928 | 0.0581 | 0.0022 | 0.0016 | 0.0006 | Reg |
| | 1.1500 | 0.0891 | 0.1094 | 0.0783 | 0.0485 | 0.0001 | 0.0022 | 0.0003 | Mom |
| | 1.4500 | 0.1734 | 0.1101 | 0.0639 | 0.0389 | 0.0008 | 0.0024 | 0.0036 | Mom |
| | 1.7500 | 0.2578 | 0.1108 | 0.0494 | 0.0293 | 0.0043 | 0.0001 | 0.0004 | Mle |
| | 2.0500 | 0.3422 | 0.1115 | 0.0349 | 0.0197 | 0.0106 | 0.0035 | 0.0108 | Mle |

Table (4.4) estimator fuzzy hazard rate when ( c=3, b=3.2, ʒ=0.8, ǩ =0.6)

| n | $t_i$ | h(t) | $\hat{h}_{mom}$ | $\hat{h}_{mle}$ | $\hat{h}_{reg}$ | Mse-mom | Mse-mle | mse-reg | Best |
|---|---|---|---|---|---|---|---|---|---|
| 40 | 0.8500 | 0.2438 | 0.0836 | 0.1230 | 0.0589 | 0.0051 | 0.0029 | 0.0068 | Mle |
| | 1.1500 | 0.4125 | 0.0843 | 0.0517 | 0.0223 | 0.0215 | 0.0260 | 0.0304 | Mom |
| | 1.4500 | 0.5813 | 0.0851 | 0.0197 | 0.0142 | 0.0492 | 0.0631 | 0.0643 | Mom |
| | 1.7500 | 0.7500 | 0.0858 | 0.0911 | 0.0507 | 0.0882 | 0.0887 | 0.0978 | Mom |
| | 2.0500 | 0.9187 | 0.0865 | 0.1624 | 0.0873 | 0.1385 | 0.1144 | 0.1383 | Mle |
| 60 | 0.8500 | 0.2438 | 0.0964 | 0.1236 | 0.0735 | 0.0043 | 0.0029 | 0.0058 | Mle |
| | 1.1500 | 0.4125 | 0.0974 | 0.0528 | 0.0279 | 0.0199 | 0.0259 | 0.0296 | Mom |
| | 1.4500 | 0.5813 | 0.0984 | 0.0180 | 0.0177 | 0.0630 | 0.0466 | 0.0635 | Mle |
| | 1.7500 | 0.7500 | 0.0994 | 0.0889 | 0.0634 | 0.0847 | 0.0874 | 0.0943 | Mom |
| | 2.0500 | 0.9187 | 0.1004 | 0.1597 | 0.1090 | 0.1339 | 0.1132 | 0.1311 | Mle |
| 80 | 0.8500 | 0.2438 | 0.1117 | 0.1243 | 0.0545 | 0.0035 | 0.0029 | 0.0046 | Mle |
| | 1.1500 | 0.4125 | 0.1131 | 0.0614 | 0.0207 | 0.0179 | 0.0247 | 0.0286 | Mom |
| | 1.4500 | 0.5813 | 0.1144 | 0.0014 | 0.0132 | 0.0436 | 0.0626 | 0.0631 | Mom |
| | 1.7500 | 0.7500 | 0.1158 | 0.0643 | 0.0470 | 0.0804 | 0.0868 | 0.0934 | Mom |
| | 2.0500 | 0.9187 | 0.1172 | 0.1271 | 0.0808 | 0.1285 | 0.1123 | 0.1304 | Mle |

## Results and Discussion:

In table(1) include four sub tables due to different sets of initial values of parameters $(b,\ c, \delta, \check{k})$, and all the results of estimation are compared used Mean square error (MSE), so we find that Moments estimator(MOM), is best for all inputs table (1), with percentage (51%), while (MLE) estimators of fuzzy hazard rate function is preferred with (28%), while the third fuzzy hazard estimator is obtained from regression estimation with percentage (21%).

For main table (2) and according to different results for estimating $\hat{h}(t_i)$, we conclude $\hat{h}(t_i)_{mom}$is best with percentage (38%), and the $\hat{h}(t_i)_{mle}$ is best with (48%), and the third one is $\hat{h}(t_i)_{reg}$is best with (14%).

Finally for third Main table (3) and according to different results for estimating fuzzy hazard rate function $\hat{h}(t_i)$, by three different methods, we conclude the $\hat{h}(t_i)_{mom}$ is best with (36%), while $\hat{h}(t_i)_{mle}$ is best (45%), and $\hat{h}(t_i)_{reg}$ is best with (19%).

And finally for table (4), the results indicates that $\hat{h}(t_i)_{mom}$ is best with (43%) and $\hat{h}(t_i)_{mle}$ is best with (43%), finally $\hat{h}(t_i)_{reg}$ is best with (14%).

Finally the best estimators of $\hat{h}(t_i)$ is $\hat{h}(t_i)_{mle}$, between all alternative, due to its excellent properties, that is satisfied by (MLE) estimators.

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